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Question Paper Code : 51573

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2014.

Fourth Semester

Electronics and Communication Engineering

MA 2261/MA 45/MA 1253/080380009/10177 PR 401 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulation 2008/2010)

Time : Three hours — Maximum : 100 marks

(Use of statistical tables is permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. X and Y are independent random variables with variance 2 and 3. Find the variance of $3X + 4Y$.

2. A continuous random variable X has probability density function (pdf) $f(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0; & otherwise \end{cases}$. Find k such that $P(X > k) = 0.5$.

3. State Central Limit Theorem for iid random variables.

4. State the basic properties of joint distribution of (X, Y) when X and Y are random variables.

5. State the properties of an ergodic process.

6. Explain any two applications of a binomial process.

7. Define Cross-correlation function and state any two of its properties.

8. Find the variance of the stationary ergodic process $\{X(t)\}$ whose auto correlation function is given by $R_{XX}(\tau) = 25 + 4/(1 + 6\tau^2)$.

9. Define a system. When is it called a linear system?

10. Define Band-Limited white noise.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Define the moment generating function (MGF) of a random variable? Derive the MGF, mean, variance and the first four moments of a Gamma distribution. (8)

(ii) Describe Binomial B (n, p) distribution and obtain the moment-generating function. Hence compute (1) the first four moments and (2) the recursion relation for the central moments. (8)

Or

(b) (i) A random variable X has the following probability distribution.

X:	0	1	2	3	4	5	6	7
$P(x)$:	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

Find

(1) the value of K.

(2) $P(1.5 < X < 4.5 / X > 2)$ and

(3) The smallest value of n for which $P(X \le n) > \frac{1}{2}$. (8)

(ii) Find the MGF of a random variable X having the pdf $f(x) = \begin{cases} \dfrac{x}{4e^{+x/2}} & x > 0 \\ 0; & elsewhere \end{cases}$. Also deduce the first four moments about the origin. (8)

12. (a) If the joint pdf of a two dimensional random variable (X, Y) is given by

$$f(x, y) = \begin{cases} x^2 + \dfrac{xy}{3}, & 0 < x < 1;\ 0 < y < 2 \\ 0, & otherwise \end{cases}.$$

Find

(i) $P\left(X > \dfrac{1}{2}\right)$

(ii) $P(Y < X)$

(iii) $P[X + Y \ge 1]$ and

(iv) Find the conditional density functions. (16)

Or

(b) (i) The joint p.d.f. of the random variable (X, Y) is $f(x, y) = 3(x + y)$ $0 \le x \le 1, 0 \le y \le 1, x + y \le 1$, find Cov (X, Y). (8)

(ii) Marks obtained by 10 students in Mathematics (x) and statistics (y) are given below :

x:	60	34	40	50	45	40	22	43	42	64
y:	75	32	33	40	45	33	12	30	34	51

Find the two regression lines. Also find y when $x = 55$. (8)

13. (a) (i) The process $\{X(t)\}$ whose probability distribution under certain condition is given by $P\{X(t) = n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2 \\ \dfrac{at}{1+at}, & n = 0 \end{cases}$. Find the mean and variance of the process. Is the process first-order stationary? (10)

(ii) If the WSS process $\{X(t)\}$ is given by $X(t) = 10\cos(100t + \theta)$, where θ is uniformly distributed over $(-\pi, \pi)$, prove that $\{X(t)\}$ is correlation ergodic. (6)

Or

(b) (i) If the process $\{X(t); t \ge 0\}$ is a Poisson process with parameter λ, obtain $P\{X(t) = n\}$. Is the process first order stationary? (10)

(ii) Prove that a random telegraph signal process $Y(t) = \alpha X(t)$ is a Wide Sense Stationary Process where α is a random variable which is independent of $X(t)$ and assumes values -1 and +1 with equal probability and $R_{XX}(t_1, t_2) = e^{-2\lambda|t_1 - t_2|}$. (6)

14. (a) (i) Find the mean and auto correlation of the Poisson process. (8)

(ii) Prove that the random processes $X(t)$ and $Y(t)$ defined by $X(t) = A \text{ Cos } wt + B \text{ Sin} wt$ and $Y(t) = B \text{ Cos } wt - A\, B \text{ Sin} wt$ are jointly wide sense stationary. (8)

Or

(b) State and prove Weiner-Khintchine Theorem. (16)

15. (a) (i) Show that if the input $\{X(t)\}$ is a WSS process for a linear system then output $\{Y(t)\}$ is a WSS process. Also find $R_{XX}(\tau)$. (8)

(ii) If $\{X(t)\}$ is the input voltage to a circuit and $\{Y(t)\}$ is the output voltage, $\{X(t)\}$ is a stationary random process with $\mu_X = 0$ and $R_{xx}(\tau) = e^{-\alpha|\tau|}$. Find the mean μ_Y and power spectrum $S_{YY}(\omega)$ of the output if the power transfer function is given by $H(\omega) = \dfrac{R}{R + iLW}$. (8)

Or

(b) (i) If $Y(t) = A\cos(\omega t + \theta) - N(t)$, where A is a constant, θ is a random variable with a uniform distribution in $(-\pi, \pi)$ and $\{N(t)\}$ is a band limited Gaussian white noise with power spectral density

$$S_{NN}(\omega) = \begin{cases} \dfrac{N_0}{2}, & for\ |\omega - \omega_0| < \omega_B \\ 0, & elsewhere \end{cases}.$$

Find the power spectral density $Y(t)$. Assume that $\{N(t)\}$ and θ are independent. (10)

(ii) A system has an impulse response $h(t) = e^{-\beta t}U(t)$, find the power spectral density of the output $Y(t)$ corresponding to the input $X(t)$. (6)